NUMERICAL SOLUTION OF INVERSE NONLINEAR PROBLEMS IN UNSTEADY HEAT CONDUCTION

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UDC 536.246:517.949.8

Results are presented for solution of inverse heat-conduction problems, solved by a trial-and-error method using analog and digital computers (implicit scheme, mesh method).

In recent years inverse problems (IP) of heat conduction have received increasing attention, since in many cases solution of IP allows one to define boundary conditions more simply than by other methods. Sometimes the IP solution is a unique source of information as to boundary conditions for an actual structure. The bibliography of papers on IP heat conduction already numbers several hundred items. A survey and a classification of methods of heat-conduction IP solution will be the subject of a separate publication, and here we will only address recent papers [1-4], which gave a short bibliography on the application of approximate analytical and numerical methods; direct and regularizing methods.

Analog computers (AC), which are used for various numerical methods of solution of direct problems, using trial and error, began to be applied to the solution of heat-conduction IP about 20 years ago [5], and with each passing year they find increasing application [6-8]. The error in determining heat fluxes or heat-transfer coefficients in solution of IP, other conditions being equal, depends considerably on the error of the initial data (ΔT). The error is defined as the difference between the temperature given from the experiment (physical or numerical) (T_e) and the "true" temperature value (T),

$$\Delta T = T_e - T.$$

We use the expression "true" temperature for the value obtained by exact analytical, approximate analytical, or numerical methods. In our case the true temperatures are obtained by numerical solution of the direct problem. The accuracy of this kind of numerical solution resides in the assigned limits (e.g., ±0.05°) and was checked by special investigations similar to the present one, as described in [8-14].

The trial-and-error, or choice, method which uses a numerical method (in our case a mesh method, implicit finite-difference scheme), has a source of errors not only in ΔT, but also in other factors. The effect of these factors on the accuracy of solution of direct problems by numerical methods on an analog or digital computer has been investigated, for example, in [8-14]. Since a direct method of solution of heat-conduction problems is used in the trial-and-error method, one must investigate the effect of specific errors of the method; for example, the errors in ΔT and the algorithm program errors (ε_a). The trial-and-error method involves operations using the method of solution of the corresponding direct problem, comparison operations, and operations to vary parameters until one satisfies the condition

$$|\bar{T}_e - T| \leqslant \varepsilon, \tag{1}$$

where ε is the error of the method. Thus, the trial-and-error method assumes that there is a certain initial error in the input data. The numerical experiment allows us to choose ε so that

$$\varepsilon = \varepsilon_a + \Delta T.$$

Results of interesting investigations of the effect of the value of ε on the accuracy of solving heat-conduction IP were given in [2], where the author succeeded in applying the method of regularization proposed by Tikhonov. Below we investigate the error of the trial-and-error method, and the results are characterized by the fact that, in contrast with what was done in [2, 4], no smoothing or regularization was done for

Institute of Engineering Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 6, pp. 1076-1080, June, 1975. Original article submitted October 17, 1973.

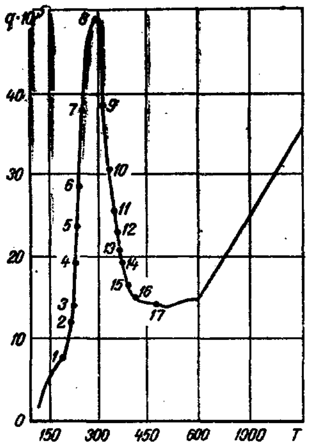

Fig. 1. The heat flux (kcal/$m^2 \cdot h$) as a function of temperature (deg) of the surface during boiling in water. The numbers denote values of q and T_p, for which the IP was solved (see Figs. 2 and 3).

the original experimental data. It would seem that smoothing and/or regularization must reduce the error in solving an IP, since these operations would otherwise be useless. It is interesting, particularly in automating the experiment and in reducing the data obtained, to analyze the errors of an IP without preliminary smoothing and/or regularization.

The mathematical model of the original problem can be represented in the form

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - c_V \frac{\partial T}{\partial \tau} = 0 \ (0 < x < L), \tag{2}$$

$$-\lambda \frac{\partial T}{\partial x} = q \ (x = 0), \tag{3}$$

$$\frac{\partial T}{\partial x} = 0 \ (x = L), \tag{4}$$

$$T(x, 0) = T_{\max}, \tag{5}$$

where λ, $c_V = c_p$, q are functions of temperature, i.e., the problem is nonlinear. In the case when c_V is the effective heat capacity, including structural heat of internal transformations, the general problem is nonlinear, with nonlinearities of types I, II, and III [12]. In inverse problems the unknowns are the boundary conditions (i.e., q or α), and in reverse problems they are λ, c_V*; and in transformed problems the unknowns are the previous temperature distributions, while the next distributions are known. Some results are presented below for solution of a nonlinear inverse problem, where the dependence of surface heat flux on temperature (see Fig. 1) was determined for boiling in a large volume. The points on Fig. 1 denote values of q and α, for which the nonlinear IP was solved on an analog computer, a network integrator, and on the MIR-1 digital computer. The true temperature field was assumed to be that obtained by solving the nonlinear direct problems by a mesh method, using an implicit scheme. Analysis of the effect of all the factors governing the accuracy of solution of a direct nonlinear problem allows the conclusion to be made that these fields are reference quantities for solution of an IP. On both types of computer the

*Examples of solution of inverse and reverse problems on electrical models are given, for example, in [8].

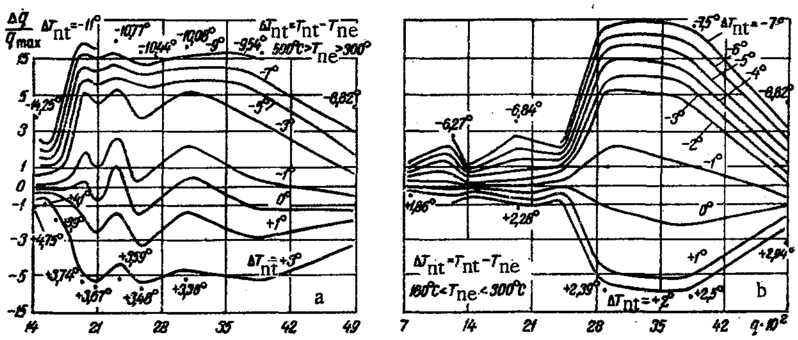

Fig. 2. Relative errors in heat flux as a function of the experimental error and the heat flux value (a) for T_p = 500-300°C; b) for T_p = 300-180°C). $\Delta q/q_{max}$ is in %, q is in kcal/$m^2 \cdot h$.

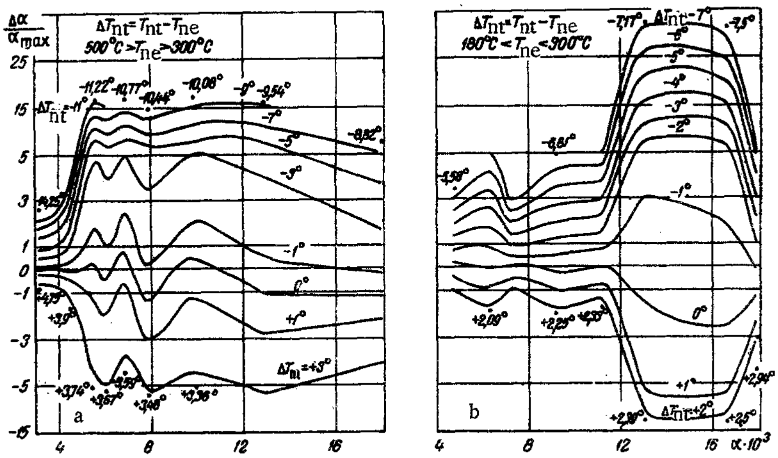

Fig. 3. Relative errors in heat-transfer coefficients as a function of the experimental error and the heat transfer coefficient for $T_{medium} = 20°C$ (a) for $T_p = 500\text{-}300°C$; b) for $T_p = 300\text{-}180°C$). $\Delta\alpha/\alpha_{max}$ is in %, α is in kcal/m^3 · deg.

trial-and-error method includes the use of a numerical mesh method (implicit finite-difference scheme). The Libmann method [8, 9] was used on the analog computer, and a well-known calculation routine in implicit scheme was used on the digital computer to calculate nonlinearities in a noniterative scheme [15].

Figures 2 and 3 show the dependence of errors in heat flux (heat-transfer coefficient) on the heat flux values (heat-transfer coefficients) for various values of ε. The algorithm error is $\varepsilon_a = 0.1°C$. Thus, the total errors are equal to the values of ΔT shown in the figures plus ε_a. The effect of the error $\varepsilon_a = 0.1°C$ can be seen from the curves for $\Delta T = 0$, for which $\varepsilon = \varepsilon_a$. It can be seen in the figures that the errors in q and α depend appreciably on ΔT. However, for specific conditions a solution of the IP by the trial-and-error method can be found without smoothing and regularization. The errors in q and α depend on the ratios of the thermal resistances determining the governing heat fluxes, which depend, in turn, on heat conduction, heat capacity, and on the external source of heat. In stationary linear IP it is comparatively simple to evaluate the error; it depends on the values of Bi or Ki, i.e., the ratio of the external thermal resistance (R_α or R_q) and the thermal resistance of heat conduction (R_λ). In unsteady nonlinear IP the error should be evaluated for each specific case by the trial-and-error method on an analog or digital computer. To investigate the effect of all the factors it is desirable to use the method of scientific planning of an experiment, since the traditional approach, i.e., to investigate the effect of each factor at several levels with constant values of the other factors, requires several thousand calculations.

It can be seen from Figs. 1-3 that the errors Δq and $\Delta\alpha$ affect dq/dT or $d\alpha/dT$, and this must be taken into account in solving a heat-conduction IP.

Finally, we discuss two aspects of the work of [14], one of which is closely associated with the analysis of accuracy of solution of inverse problems. Firstly, numerical solution of a direct nonlinear problem permits new physical phenomena to be revealed. A clear example is the T-layer effects discovered by Samarskii et al. Secondly, numerical solution of a direct nonlinear problem is often the only source of a reference "true" solution to serve as a basis for development of a method of solving the inverse problem.

The oscillations in surface temperature, noted in [14], could be assumed to be the "discovery" of a physical effect investigated experimentally and theoretically for similar but not identical conditions [16, 17]. Special investigations that we have made have shown that in the case described in [14], the oscillations were caused by the scheme for computing the nonlinearity. If we calculate the effect of nonlinearities by an appropriate method (automatic choice of the time interval), the oscillations obtained in [14] vanish. Thus, the fact noted in [14] that there is reduction of amplitude and frequency of oscillation with reduction of the time interval is confirmed, and detailed investigations have shown how one can avoid the appearance of such oscillations in numerical solutions of nonlinear problems.

Thus, careful analysis of the effect of these factors in solving both direct and inverse nonlinear problems allows us not only to avoid erroneous "discovery," but also to obtain reliable data for reference solutions. In the case where there are surface oscillations, as in [16, 17], one must make a careful analysis of the routine for solution of the inverse problem. The temperature oscillations obtained in thermal experiments in [17] reflect oscillations of real actual heat flux at the surface. These oscillations of q may be perceived quite accurately when the trial and error method is performed on electrical models or on digital computers.

A numerical experiment to solve nonlinear IP of unsteady heat conduction on analog and digital computers shows that for specific requirements as to accuracy of the experimental data, the trial-and-error method can be used without smoothing and regularization to determine boundary conditions in the transition regimes. This conclusion may be useful, since it simplifies the method and technique for automating a thermal experiment for which one objective is to solve the IP.

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